Beginner’s Algebra
Careful reading and template-based reasoning

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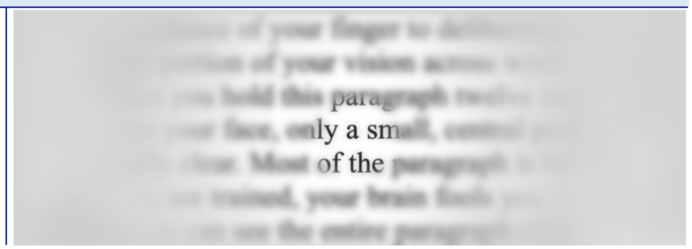
Use this packet to develop reading and reasoning skills that are useful during Algebra 1. Throughout this document, “read” means pointing and reading aloud:

Point as you read aloud 指差喚呼

Use the guidance of your finger to deliberately drag the central portion of your vision across words as you read. When you hold this paragraph twelve inches away from your face, only a small, central portion of it is actually clear. Most of the paragraph is blurry. Unless you are trained, your brain fools you into thinking you can see the entire paragraph clearly.



Convert words into a diagram

Translate the following passage into pictures or tables that are easy for people who don't read English to understand.

Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?

Reading steps

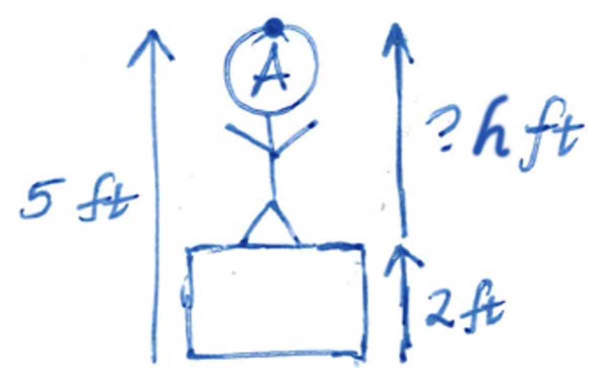
1. **Read** a short phrase containing one of the following items (occasionally, a phrase has multiple roles).
    a. **Object/person** (possibly with specification of characteristics)
    b. **Action** (possibly with specification of styles/manners in which action is carried out)
    c. **Location/time**
    d. **Quantity** (dimension or count)
2. **Draw** a simple representation of the phrase (unless the meaning of the phrase has already been sketched).
3. **Underline** the phrase. If no additional sketching was needed to illustrate the phrase, dash-underline the phrase.
4. Analyze **next** phrase.

Table 1. Breaking passage into short phrases

	Phrase	Main features	Sketch
1.	Alice	Object/person	Stick figure; capital "A" in face
2.	stands	Action	No additional sketching needed: Alice's stick figure was already drawn upright.
3.	on a box	Location/time	Rectangle immediately under Alice
4.	that has a height of 2 feet.	Quantity	Upward arrow from bottom to top of rectangle labeled "2 ft"
5.	The top of Alice's head	Object/person	Dot marking top of Alice's stick figure
6.	is 5 feet above the ground.	Quantity and Location/time	Upward arrow from bottom of rectangle to dot marking top of Alice labeled "5 ft"
7.	How tall is Alice?	Quantity (requested)	Upward arrow from bottom of Alice to dot marking top of Alice labeled "? *h* ft"

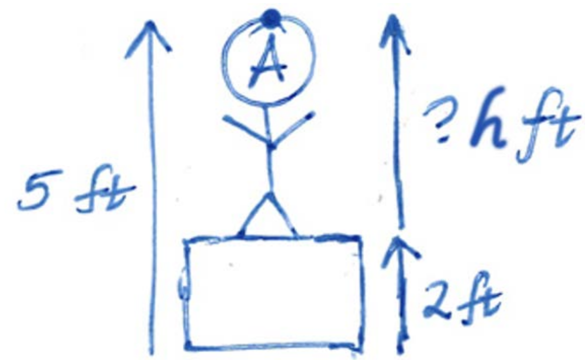
Showing the table above is usually unnecessary. Underlining the printed problem statement and drawing a sketch usually suffices.

Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?



Convert a diagram into an equation and solve

The problem statement and sketch from the previous page are copied below and to the right.



> Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?

Answer the question.

Steps for converting a word problem's diagram into an equation and solving

1. Look for a starting equation.
    a. If there's a flashcard with a relevant equation, check that conditions the flashcard lists are met. If so, copy the equation.
    b. If no suitable flashcard equation is available, create a starting equation.
        i. Use your finger to point at and use your words to narrate a collection of related diagrammed quantities (Table 2, columns A and B).
        ii. Identify quantities, operations, and relations in your narration (Table 2, columns B, C, and D).
        iii. Algebraically abbreviate each quantity, operation, and relation (Table 2, columns C, D, and E).
2. Use rules of algebra (see pages 5 and 6) to isolate the variable whose value is to be found.
3. Write a sentence presenting the value you found, with appropriate sign ($\pm$) and units.

Table 2. Converting a diagram into an equation and solution

A. Gesture	B. Narrate	C. Quantities, operations, and relations	D. Category	E. Algebra
Trace finger up across arrow indicating the box's height	"Ascending through the 2-foot height of the box"	"Ascending through the 2-foot height of the box"	Quantity	2
Trace finger up across arrow indicating Alice's height	"and then further ascending through Alice's unknown height h"	"and then further ascending through"	Operation	$+$
		"Alice's unknown height h"	Quantity	h
Trace finger up across arrow from bottom of box to top of Alice's head	"travels through the same height change as ascending 5 feet from the bottom of the box to the top of Alice's head."	"travels through the same"	Relation	$=$
		"height change as ascending 5 feet from the bottom of the box to the top of Alice's head."	Quantity	5

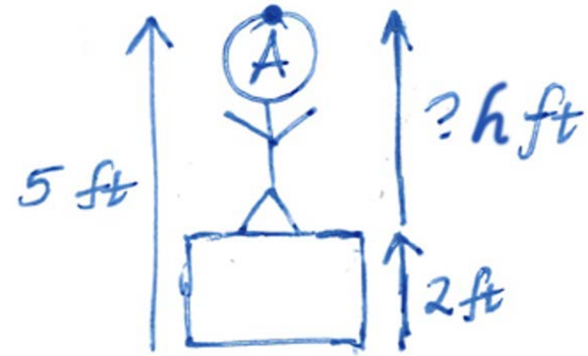
Starting equation: $2 + h = 5$

See algebraic solution and summary sentence on next page.

Table 3. Algebraically solve (see pages 5 and 6 for technique):

A. Work you show	B. Name of template
$2 + h = 5$	
$\cancel{a}\,2+h = \cancel{b}\,5$ $\downarrow$ $\cancel{a}\,2+h = \cancel{b}\,5$ $-\cancel{c}\,2 \quad -\cancel{c}\,2$ $\cancel{a}\,2+h - \cancel{c}\,2 \quad \cancel{b}\,5 - \cancel{c}\,2$	S. Subtraction Property of Equality
$2 + h - 2 = 5 - 2$	
$\cancel{a}\,2 + \cancel{b}\,h$ $\Updownarrow$ $\cancel{b}\,h + \cancel{a}\,2$	E. Commutative Property of Addition
$h + 2 - 2 = 5 - 2$	
$\cancel{a}\,2 - \cancel{a}\,2$ $\Updownarrow$ 0	N. Inverse Property of Addition
$h + 0 = 5 - 2$	
$\cancel{a}\,h + 0$ $\Updownarrow$ $\cancel{a}\,h$	J. Identity Property of Addition
$h = 5 - 2$	
$h = 3$	Arithmetic OK to use calculator to evaluate $5 - 2$

Summarize: “Alice’s height is 3 feet.”



Carry out and narrate a step of algebraic reasoning

Table 4. Showing and describing algebraic steps.

Step	**Algebraic reasoning**	**Narration** (usually not requested)
State **question**.	Simplify $3x + 4x$.	Simplify $3x + 4x$.
Copy **template** from notes or textbook	$ac + bc$ $\downarrow$ $(a + b)c$	**By** the distributive property, $ac + bc$ can be replaced by $(a + b)c$.
Replace template symbols with symbols from current discussion.	$\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$	**Regard** 3 as a, 4 as b, and x as the common factor c.
State **claim** and clean up.	$(3 + 4)x$ $7x$	**So,** $3x + 4x$ can be replaced by $(3 + 4)x$, in other words, $7x$.
Finished example	Simplify $3x + 4x$. $\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$ $(3 + 4)x$ $7x$	Simplify $3x + 4x$. **By** the distributive property, $ac + bc$ can be replaced by $(a + b)c$. **Regard** 3 as a, 4 as b, and x as the common factor c. **So,** $3x + 4x$ can be replaced by $(3 + 4)x$, in other words, $7x$.

Basic algebra templates

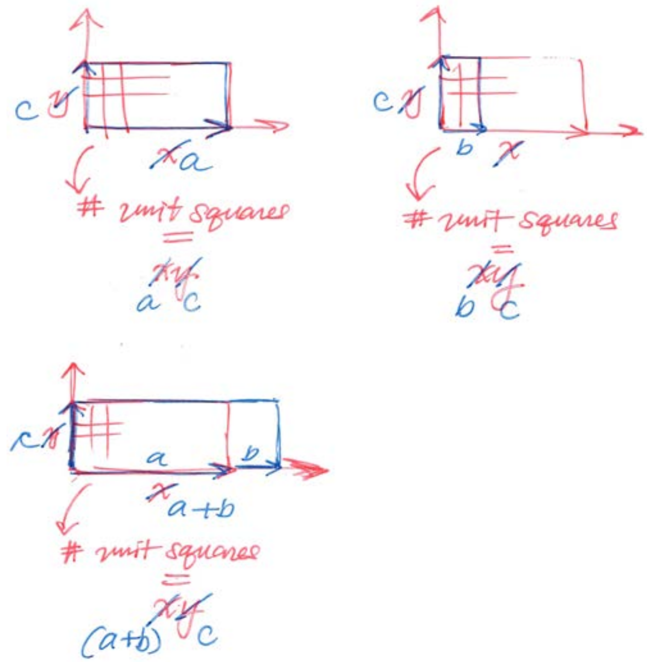
For exercises, see corresponding "EA" sections in Marecek *et al.*, *Elementary Algebra 2e*, available for free through a CC BY license at OpenStax: https://openstax.org/details/books/elementary-algebra-2e

Table 5. Basic algebra templates

EA	Name	Template(s)	
1.9	**A.** Division by zero is undefined	Any denominator or factor in denominator = 0 $\downarrow$ Discard template	
1.5	**B.** Product of fractions	$\frac{a}{b}\cdot\frac{c}{d}$ $\updownarrow$ $\frac{ac}{bd}$	
1.5	**C.** Canceling factors in numerator & denominator	$\frac{a\cdot c}{b\cdot c}$ $\updownarrow$ $\frac{a}{b}$	$\frac{c\cdot a}{c\cdot b}$ $\updownarrow$ $\frac{a}{b}$
1.5	**D.** Quotient of fractions	$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}$ $\updownarrow$ $\frac{a}{b}\cdot\frac{d}{c}$	
1.9	**E.** Commutative Property of Addition	$a+b$ $\updownarrow$ $b+a$	
1.9	**F.** Commutative Property of Multiplication	$a\cdot b$ $\updownarrow$ $b\cdot a$	
1.9	**G.** Associative Property of Addition	$a+b+c$ $\updownarrow$ $(a+b)+c$ $\updownarrow$ $a+(b+c)$	
1.9	**H.** Associative Property of Multiplication	$a\cdot b\cdot c$ $\updownarrow$ $(a\cdot b)\cdot c$ $\updownarrow$ $a\cdot(b\cdot c)$	
1.9	**I.** Distributive Property	$a\cdot(b+c)$ $\updownarrow$ $a\cdot b+a\cdot c$ $a\cdot(b-c)$ $\updownarrow$ $a\cdot b-a\cdot c$	$(b+c)\cdot a$ $\updownarrow$ $b\cdot a+c\cdot a$ $(b-c)\cdot a$ $\updownarrow$ $b\cdot a-c\cdot a$
		$(a+b)\cdot(c+d)$ $\updownarrow$ $\begin{array}{c\|c\|c} & c & d \\ \hline a & a\cdot c & a\cdot d \\ \hline b & b\cdot c & b\cdot d \end{array}$ $a\cdot c+b\cdot c+a\cdot d+b\cdot d$	
1.9	**J.** Identity Property of Addition	$a+0$ $\updownarrow$ a	$0+a$ $\updownarrow$ a
1.9	**K.** Identity Property of Multiplication	$a\cdot 1$ $\updownarrow$ a	$1\cdot a$ $\updownarrow$ a
1.9	**L.** Identity Property of Division	$\frac{a}{1}$ $\updownarrow$ a	
1.9	**M.** Adding additive inverse is equivalent to subtracting	$a+(-b)$ $\updownarrow$ $a-b$	
1.9	**N.** Inverse Property of Addition	$a+(-a)$ $\updownarrow$ 0	$a-a$ $\updownarrow$ 0
1.5 & 1.9	**O.** Inverse Property of Multiplication (and the related One Property of Division)	$a\cdot\frac{1}{a}$ $\updownarrow$ 1	$\frac{a}{a}$ $\updownarrow$ 1
1.9	**P.** Multiplication by zero	$a\cdot 0$ $\updownarrow$ 0	$0\cdot a$ $\updownarrow$ 0
1.9	**Q.** Division involving zero	$\frac{0}{a}$ $\updownarrow$ 0	See "Division by zero is undefined"
2.1	**R.** Addition Property of Equality	$a=b$ $\downarrow$ $\begin{array}{ccc} a & = & b \\ +c & & +c \\ \hline a+c & = & b+c \end{array}$	
2.1	**S.** Subtraction Property of Equality	$a=b$ $\downarrow$ $\begin{array}{ccc} a & = & b \\ -c & & -c \\ \hline a-c & = & b-c \end{array}$	
2.2	**T.** Multiplication Property of Equality	$a=b$ $\downarrow$ $a\cdot c=b\cdot c$	
2.2	**U.** Division Property of Equality	$a=b$ $\downarrow$ $\frac{a}{c}=\frac{b}{c}$	
6.2	**V.** Exponent notation	a^2 $\updownarrow$ $a\cdot a$	a^n $\updownarrow$ $\underbrace{a\cdot a\cdots a}_{n\text{ copies}}$
7.6	**W.** Zero-Product Property	$a\cdot b=0$ $\downarrow$ $a=0$ or $b=0$	
9.1/9.7	**X.** Root notation	$\sqrt[n]{a}=b$, even n $\updownarrow$ $b^n=a, b\geq 0$, even n	$\sqrt[n]{a}=b$, odd n $\updownarrow$ $b^n=a$, odd n
9.8*	**Y.** A correction to the Power Property	$\sqrt{a^2}$ $\updownarrow$ $\|a\|$	
10.3	**Z.** Quadratic Formula	$ax^2+bx+c=0$ $\updownarrow$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	

“Derive” an algebraic rule from a graphical representation

Table 6. Using multiple representations when justifying an algebraic rule

Narration of template	**Algebraic template**	**Graphical justification**
By the distributive property, $ac + bc$ can be replaced by $(a + b)c$.	$ac + bc$ $\downarrow$ $(a + b)c$	

Multiplication of real numbers is graphically represented by drawing a 2-dimensional coordinate system, with a horizontal arrow from the origin along the horizontal axis representing the real number x and a vertical arrow from the origin along the vertical axis representing the real number y. A rectangular region with the x- and y- arrows as two sides is partitioned into unit squares. A caption emanating from the rectangular region is labeled number of unit squares = xy. This graphical template is drawn three times. In the first, the letters x and y are slashed out and replaced by, respectively, a and c. In the second, the letters x and y are slashed out and replaced by, respectively, b and c. In the third, the letters x and y are slashed out and replaced by, respectively, $a + b$ and c. The horizontal arrow labeled $a + b$ is broken into two pieces, labeled a and b. The horizontal arrows labeled a are congruent, and the horizontal arrows labeled b are congruent. The number of unit squares representing the product ac plus the number of unit squares representing the product bc equals the number of unit squares representing the product $(a + b)c$, so $ac + bc$ can be replaced by $(a + b)c$.